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WHAT IS MATHEMATICS?

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What is mathematics? My brief answer is, Invention, Logic, and naming aspects of what Bernard Lonergan calls the empirical residue which is left unexplained when we have discovered the intelligibility of the data of our experience.

The logical component of mathematics is well known: a range of deductions from axioms or postulates. Euclid is famous for a most remarkable achievement of such a logical structure in his geometry, though we must remember that there were great geometers before his time, whose inventive work was rather overshadowed by his systematisation. Sometimes, especially about one century ago, it has been claimed that the whole of mathematics is logic. I will concentrate here on other components that are more important, and less often celebrated.

Learning mathematics is not easy, and often succumbs, more seriously than in other subjects, to the obstacles that occur even in schools and with well-trained teachers. So my account has to begin with an explanation of what we are doing when we are learning, the question put by Lonergan that leads on to his structure of learning, that is, of coming to know.

We can approach his answer, expressed in three levels with two or three activities on each level, by attending to our own learning activities, especially the wonder, or spirit of inquiry, or desire to know that motivates them all. This wonder we express as questions and we satisfy, to some extent, with answers. Questions, though dependent on the drive of wonder from which they stem, are a very important part of our language, signalled by a dozen or so easily recognizable words, such as Where? When? Which? Who? What? How? Why? along with such inversions as Is it? Did they? Can we? Aristotle tried in his earliest work to classify our answers according to ten of these, but later claimed that there were only four types of question and then promptly reduced these to two. We can recognize those two types of answer (to which I will later add two more, though at first sight, we might not quite look on these as answers) fairly easily by pointing out that there is one type of answer that is normally extremely short and another that needs to be longer, often indeed very much longer.

The short answers are Yes, and No. These indicate, in slightly different ways, the completion of one incident of learning, especially when the question (such as Is that really so?) came directly from our own wonder rather than that of some other person. The answer Yes (called an Assent or an affirmation) requires a personal satisfaction (which I call Judging) that the available evidence justifies such a definitive conclusion that I would be silly to add "I think" or "Perhaps" or any of the hundred other ways in which we are, so often, quite right to hedge. Judging and Assent form the third, or top, level of learning, or of what Lonergan calls the structure of knowing. Are you now reading what claims to be an account of the learning process? Yes? Is it reasonable for you to give such an assent?

The long answer to a question, on the contrary, is presented as an explanation, a supposition, a possibility, in a proposition or a set of propositions, all based on one's own understanding, a discovery or insight or invention that may have been prepared entirely by one's own investigation and puzzling, but more frequently is facilitated by a formal or informal teacher who has arranged the data that enables us to grasp, at length, the point. In any grade six class there is usually at least one student who can occupy a few minutes in answering intelligently the question, What is a motor car? These activities of understanding and of expressing one's understanding form the second level of Lonergan's structure of coming to know. Intelligent relations between formal propositions expressing our insights form most of the material of logical discourse, which fits therefore on this second level, while leading to the demand for a definitive assent.

A long answer giving an explanation and a short answer like Yes correspond to the two types of questions to which Aristotle said all questions can be reduced, namely What is an X? (which includes Why is X a Y?) and Is there an X? (which includes Is X a Y?). But the question What is a motor car? presupposes that we have some experience of motor cars, for instance by hearing them, seeing them moving, sitting in them, and perhaps turning a steering wheel and pushing an accelerator. Such experiences, while not exactly answers to questions, can be facilitated by another set of questions, Where? When? Which? And Who? which direct our attention to places, times, and distinct individuals, in which we might contact data for insights. Where and when can we find a motor car? When we do find one we may notice that it is moving or making a noise and so go on to ask why it does so. Whatever comes through the senses like this is called empirical data. Such data are "given" to the senses. (Lonergan extends this notion to include all that is given in consciousness, coining the term "generalized empirical method" for attempts to understand such data, parallel to the "empirical method" used by the sciences, but we need not be concerned about that today). Lonergan uses the term "empirical residue" for all that is left unexplained when the sciences have understood the data. He names four aspects of the empirical data:

- The *individual*: one more instance of the *same* intelligibility as before.
- The *continuum*: what is *continuous* (like a straight line), as opposed to discrete.
- Particular places and times (See *Insight*, 504.3/527.9) for his argument on these).
- *Randomness*, or non-systematic divergence of actual frequencies from a probability.

Thus we have arrived at Lonergan's structure of learning. The first (or lowest) level consists of experience—all our sensations, and also that experience of ourselves and our activities which we call consciousness and which we have all the time we are awake or dreaming. The second level (which presupposes the first level and complements it) consists of understanding, especially that fresh arrival of new understanding which we call an insight. The third level consists of assent to some of the possibilities that we had formulated on the second level, and of the judging we did to justify that assent.

I have said that there is a fourth type of question besides the three just mentioned, but it goes beyond learning. It is expressed as "So what?" or "Will I?" and is not followed by an answer but by a free decision and then normally action. Action, in general, does not lead to further knowledge of the world but instead it makes changes in the world that we had, till then, known. So we can call the full four-level structure "the structure of control". Lonergan calls it "the structure of knowing and doing", keeping terms like "the structure of knowing" for the three-level structure. The fourth level is not directly connected with mathematics, but is extremely important wherever responsibility is involved.

Now that we have outlined the structure of learning we can approach the nature of mathematics. To learn mathematics is not easy, even if some, like Pascal, have been able to do it for

